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Continuous quantum measurement of a qubit state

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Abstract. We consider a two-level quantum system (qubit) which is continuously measured by a detector. The conventional formalism, which implies the ensemble averaging, describes the gradual decoherence of the qubit state due to measurement. However, in each particular realization of the measurement process we can have the opposite effect: gradual purification of the qubit density matrix. This can be described by the recently developed Bayesian formalism suitable for individual quantum systems. The purification effect may be verified experimentally using present-day technology and can be useful for quantum computing. In particular, the decoherence of a single qubit can be suppressed using continuous measurement and the feedback loop.

The significant progress in experimental techniques during recent years as well as the active research on quantum computing have motivated renewed interest in the problems of quantum measurement, including the long-standing “philosophical” questions. In contrast to the usual case of averaging over a large ensemble of similar quantum systems, it is becoming possible to study experimentally the evolution of an individual quantum system. In this paper we consider the continuous measurement of a qubit (two-level system) state by a “weakly responding” detector which can be treated as a classical device [1]. As examples, we will discuss the following solid-state realizations of such a measurement. First, the location of a single electron in a double-quantum-dot can be measured by a nearby quantum point contact (QPC) in such a way that the QPC barrier height and, hence, the current through the detector are sensitive to the measured electron position [2, 3]. The second possible setup is a single Cooper pair box, the charge state of which is measured by a capacitively coupled single-electron transistor (SET) [4]. Finally, the flux state of a SQUID can be continuously measured by another inductively coupled SQUID [5].

The conventional approach describes the measurement process by the following equations for the qubit density matrix ρ_{ij} in the basis of “localized” states:

$$\dot{\rho}_{11} = -\dot{\rho}_{22} = -2 \frac{H}{\hbar} \text{Im} \rho_{12}, \quad (1)$$

$$\dot{\rho}_{12} = i \frac{\varepsilon}{\hbar} \rho_{12} + i \frac{H}{\hbar} (\rho_{11} - \rho_{22}) - \Gamma \rho_{12}, \quad (2)$$

where H is the mixing (tunneling) and ε is the energy asymmetry of the qubit states, while Γ is the dephasing due to measurement. In the case of the QPC as a detector $\Gamma = (\Delta I)^2 / 4 S_I$ [2, 3, 6] where $\Delta I = I_2 - I_1$ is the difference between the average detector currents corresponding to two localized states of the qubit (we assume $|\Delta I| \ll (I_1 + I_2)/2$) and S_I is the low frequency spectral density of the detector shot noise.

Notice that Eqs. (1)–(2) do not depend on the detector output that is a consequence of averaging over the ensemble of systems. The situation is completely different in the case of an individual quantum system since the system evolution should become dependent (“conditioned”) on the particular detector output. The theory of conditioned (selective) evolution of a pure wavefunction was developed relatively long ago, mainly for the purposes of quantum optics (see, e.g. Ref. [7] and references therein). However, for solid state structures the problem of continuous quantum measurement with an account of the measurement result has been addressed only recently [1], with the main emphasis on mixed quantum states and detector nonideality.

In the Bayesian formalism developed in Ref. [1] the evolution of the qubit density matrix ρ is described by the equations

$$\dot{\rho}_{11} = -\dot{\rho}_{22} = -2 \frac{H}{\hbar} \text{Im} \rho_{12} - \frac{2\Delta I}{S_I} \rho_{11} \rho_{22} [I(t) - I_0], \quad (3)$$

$$\dot{\rho}_{12} = i \frac{\varepsilon}{\hbar} \rho_{12} + i \frac{H}{\hbar} (\rho_{11} - \rho_{22}) + \frac{\Delta I}{S_I} (\rho_{11} - \rho_{22}) [I(t) - I_0] \rho_{12} - \gamma \rho_{12}, \quad (4)$$

where $I(t)$ is the particular detector output, $I_0 \equiv (I_1 + I_2)/2$, and γ is the extra dephasing due to the “pure environment”,

$$\gamma = \Gamma - (\Delta I)^2/4S_I. \quad (5)$$

There is no extra dephasing, $\gamma = 0$, for the measurement by a QPC, which thus represents an ideal detector, while the SET in a typical operation point is a significantly nonideal detector, $\gamma \sim \Gamma$ [4]. The SQUID can be an ideal detector only if its total sensitivity is quantum-limited [8] ($\hbar/2$ in energy units).

Equations (3)–(4) allow us to calculate the evolution of the system density matrix for a given detector output $I(t)$. In order to analyze the behavior of $I(t)$, these equations should be supplemented by the formula

$$I(t) - I_0 = \frac{\Delta I}{2} (\rho_{22} - \rho_{11}) + \xi(t), \quad (6)$$

where the zero-correlated (“white”) random process $\xi(t)$ has zero average and the same spectral density as the detector current, $S_\xi = S_I$. (We use the Stratonovich formalism for stochastic differential equations.) Notice that even though the Bayesian formalism is valid only for “weakly responding” detectors, $|\Delta I| \ll I_0$, the dimensionless coupling $\hbar\Gamma/H$ is arbitrary, so the formalism can be used in the Quantum Zeno regime as well as in the case of weakly perturbed quantum (Rabi) oscillations.

Figure 1 shows the result of the Monte-Carlo simulation of the continuous measurement by a slightly nonideal detector, $\gamma = 0.1\Gamma$, in the case when the evolution starts from the maximally mixed state, $\rho_{11} = \rho_{22} = 0.5$, $\rho_{12} = 0$. One can see that ρ_{12} gradually appears during the measurement, eventually leading to well-pronounced quantum oscillations. In the case $\gamma = 0$ the density matrix becomes pure after a sufficiently long time. This gradual purification can be interpreted as being due to the gradual acquisition of information about the system. The detector nonideality, $\gamma \neq 0$, causes decoherence and competes with the purification due to measurement.

In contrast to QPC, the SET as a detector directly affects the qubit energy asymmetry ε because of the fluctuating potential $\phi(t)$ of SET’s central island. Since there is typically a correlation between fluctuations of $I(t)$ and $\phi(t)$ [9], Eqs. (3)–(4) can be further improved

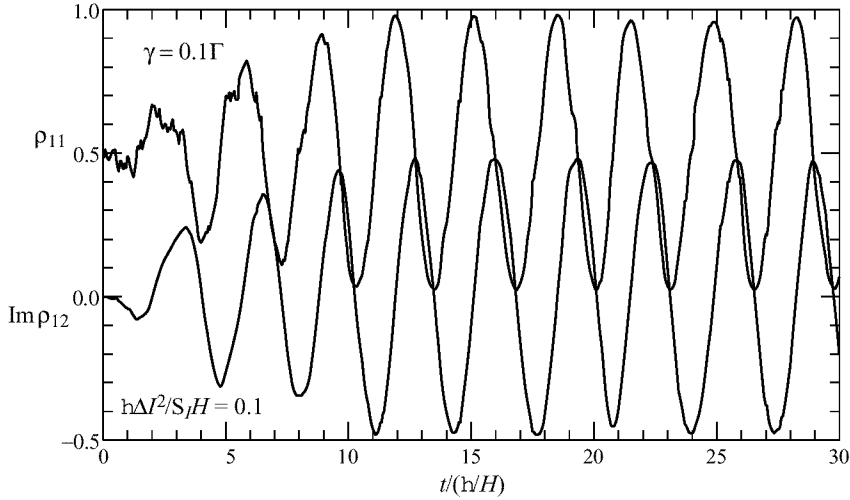


Fig. 1. Gradual purification of the qubit density matrix $\rho(t)$ in a course of continuous measurement.

taking into account the information about the likely fluctuations $\varepsilon(t)$ caused by the SET. It is natural to add into Eq. (4) the term $i\rho_{12}K[I(t) - (\rho_{11}I_1 + \rho_{22}I_2)] = i\rho_{12}K\xi(t)$, where $K = (d\varepsilon/d\phi)S_{\phi I}/S_I\hbar$ and $S_{\phi I}$ is the mutual low-frequency spectral density. One can easily check that the addition of this term corresponds to the partial recovery of coherence, so that the dephasing rate γ should be replaced with $\tilde{\gamma} = \gamma - K^2S_I/4$.

To observe the density matrix purification experimentally, it is necessary to record the detector output with sufficiently wide bandwidth, $\Delta f \gg \Gamma$ (possibly, $\Delta f \sim 10^9$ Hz), and plug it into Eqs. (3)–(4). Calculations will show the development of quantum oscillations with precisely known phase. Stopping the evolution by rapidly raising the qubit barrier ($H \rightarrow 0$) when $\rho_{11} \simeq 1$ and checking that the system is really localized in the first state, it is possible to verify the presented results.

Density matrix purification can be used in quantum computing to suppress the gradual qubit decoherence due to interaction with the environment (to keep a qubit “fresh”). The idea is to use the feedback loop which controls the qubit parameter H (control of ε is also possible) in order to decrease the difference between the desired phase of quantum oscillations and the fluctuating phase continuously monitored by a detector supplemented by the “calculator” which computes Eqs. (3)–(4). The preliminary Monte-Carlo results show very significant suppression of decoherence in the case when the detector coupling is stronger than the coupling with extra environment.

Several other predictions related to experiments with single quantum systems can also be made using the Bayesian formalism. In particular, we can show that the quantum oscillations of the qubit state can never be seen directly by the continuous detector (while they can be computed using the noisy detector output and Eqs. (3)–(4)). Quantitatively, the spectral peak of the detector output $I(t)$ at the frequency $\Omega = (4H^2 + \varepsilon^2)^{1/2}/\hbar$ of the oscillations cannot exceed $4S_I$ [10]. This is still twice as high as the classically possible limit, that is explained in the Bayesian approach by the correlation between the detector noise and the qubit evolution.

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